An Empirical Spectral Bandwidth Model for Superior Conjunction

R. S. Rockwell
Deep Space Network Operations Section

The downlink signal from spacecraft in superior solar conjunction phases suffers a great reduction in signal-to-noise ratio. Responsible in large part for this effect is the line broadening of the signal spectrum. This article presents an analytic empirical expression for spectral bandwidth as a function of heliocentric distance from 1 to 20 solar radii. The study is based on spectral broadening data obtained from the superior conjunctions of Helios 1 (1975), Helios 2 (1976) and Pioneer 6 (1968). The empirical fit is based in part on a function describing the electron content in the solar corona.

I. Introduction

During superior conjunctions the signal from a spacecraft undergoes considerable distortion as a result of passing through the solar corona. One of the prime factors responsible for this distortion is the effect of spectral broadening and is graphically seen in the degradation of the signal-to-noise ratio (SNR).

In an effort to model SNR degradation during superior conjunction it is necessary to know how spectral lines broaden as a function of distance from the Sun. This information must be obtained either from actual measurement or a theoretical model.

For the purpose of this study, Richard Woo has supplied his theoretical work on spectral broadening (Ref. 1) and a graphical display of spectral broadening data-spectral bandwidth versus heliocentric distance.

II. The Data

The data to be used in this study are a composite collection of bandwidth measurements[1] from the superior conjunctions of Helios 1 (1975), Helios 2 (1976) and Pioneer 6 (1968). The data span the region from 1 solar radius (R_0) to 20 solar radii ($20R_0$).

The Woo paper defines the bandwidth BW, as:

$$\int_0^{BW/2} P(f)df = 1/2 \int_0^{\infty} P(f)df$$

where $P(f)$ is the power spectrum of the broadened spectral line.

[1] In Hz as a function of heliocentric distance.

To utilize the bandwidth data it was decided to curve fit the raw data points in the least square sense. The immediate problem was to determine a reasonable function to be used in the fitting process.

Initially, Woo's spectral broadening paper was consulted in an effort to find a suitable "fitting" function. However, no convenient closed-form expression could be found in his work. A straightforward polynomial was considered but was considered to be too cumbersome a function and not very elegant. Next, the ISED function (integrated solar electron density) from the A. Berman/J. Wackley doppler noise formulation (Ref. 2) was investigated. There were several good reasons for considering this function.

Spectral broadening is undoubtedly in some way related to the free electron content in the solar corona. Since the ISED function is simply a measure of the electron content along the signal raypath, it seems reasonable to hypothesize that spectral broadening, which is expressed in terms of the spectral bandwidth, might be proportional to this parameter. Furthermore, the ISED parameter successfully models one solar induced effect–doppler noise. Perhaps it can another–spectral broadening.

III. The Fitting of ISED

In its original form the ISED function is given by

$$\text{ISED} = \int_0^R N_e dr$$

where

$$N(r) = \frac{A}{r^6} + \frac{B}{r^{2.3}} \frac{\text{el}}{\text{cm}^3}$$

and r is the heliocentric distance in solar radii.

The integration is along the signal path and when expressed in Sun-Earth-probe angle geometry yields:

$$\text{ISED}(\alpha, \beta) = A_0 \left[\frac{\beta}{(\sin\alpha)^{1.3}}\right] F(\alpha, \beta) + A_1 \left[\frac{1}{(\sin\alpha)^5}\right]$$

with

$$F(\alpha, \beta) = 1 - 0.05 \left[\frac{(\beta - \pi/2 + \alpha)^3 - (\alpha - \pi/2)^3}{\beta}\right] - 0.00275 \left[\frac{(\beta - \pi/2 + \alpha)^5 - (\alpha - \pi/2)^5}{\beta}\right]$$

where

α = Sun-Earth-probe angle (SEP), rad

β = Earth-Sun-probe angle (ESP), rad

When modeling doppler noise, Berman sets ISED proportional to the actual noise data and simultaneously solves for A_0, A_1 and a proportionality constant K.

In fitting the ISED function to the bandwidth data it was decided to first try using the doppler noise coefficients:

$$A_0 = 1.182 \times 10^{-3}$$

$$A_1 = 4.75 \times 10^{-10}$$

The function under consideration was of the form:

$$BW_{\text{Hz}}(\alpha) = K\ (ISED_{\text{DN}})$$

where $ISED_{\text{DN}}$ represents ISED with the doppler noise coefficients. The form of ISED was simplified slightly by setting $\beta = \pi - \alpha$. This is a reasonable approximation considering the fact that the data spans the region $1R_0$ to $20R_0$ or 0.3 to 5 deg SEP.

In performing the curve fit, the sum of the squares of the logarithmic residuals were formed

$$\sigma = \sum_{i=1}^{n} \left\{\left[\log_{10} \frac{BW_{\text{act}}(\alpha_i)}{BW_{\text{prd}}(\alpha_i)}\right]\right\}^2$$

and then minimized with respect to K:

$$\frac{\partial \sigma}{\partial K} = 0$$

This yields $K = 7.657$ with a standard deviation $\sigma = 1.6513$ dB.

The results of this fit appear to be fairly good as the statistics and a glance at the graphical data (Fig. 1) show. To improve upon this fit the next logical step was to determine a new set of coefficients, A_0 and A_1, in addition to determining K. It was also decided to look more closely at the electron density function, N_e.

Most forms of the density function found in the literature can be expressed as:

$$N_e(r) = \frac{A}{r^6} + \frac{B}{r^{2+\epsilon}}$$

where ϵ has taken on a multitude of values ranging from approximately 0.0 - 0.5. In his study, Berman chose to use an a priori value of $\epsilon = 0.3$. Berman independently derived this same value $\epsilon = 0.30$ from his curve fitting of ISED to doppler noise.

Using the same basic idea, it was decided to reevaluate ϵ in addition to A_0 for the bandwidth data. Determining the values of K, A_0, and ϵ had the same effect as determining A_1, A_0 and ϵ.

IV. Evaluation of ϵ

To evaluate the parameters A_0 and ϵ, the ISED function was written in the form:

$$\text{ISED}(\alpha, \epsilon) = A_0 \left[\frac{\pi - \alpha}{(\sin \alpha)^{1+\epsilon}}\right] F(\alpha, \epsilon) + A_1 \left[\frac{1}{(\sin \alpha)^5}\right]$$

with

$$F(\alpha, \epsilon) = 1 - \left(\frac{\epsilon}{6}\right)\left[\frac{(\pi/2)^3 - (\alpha - \pi/2)^3}{\pi - \alpha}\right] + \frac{\epsilon}{120}(3\epsilon - 2)\left[\frac{(\pi/2)^5 - (\alpha - \pi/2)^5}{\pi - \alpha}\right]$$

where the substitution $\beta = \pi - \alpha$ has been included. This is the procedure followed by Berman (Ref. 3).

Again, the least-squares method was applied and the solutions of the conditions:

$$\frac{\partial \sigma}{\partial A_0} = 0$$

$$\frac{\partial \sigma}{\partial \epsilon} = 0$$

yielded the best fit values of A_0 and ϵ. One final application of least squares determined the constant of proportionality K between the bandwidth and ISED.

The values of the ISED parameters that provide the best fit, in the least squares sense, to the bandwidth data are

$$\epsilon = 0.77$$

$$A_0 = 0.22 \times 10^{-3}$$

$$K = 8.8521$$

The corresponding statistics are

$$\sigma(\text{dB}) = 1.4347$$

This fit is plotted over the Woo data in Figs. 2 and 3 and shows the scatter between the two.

It is interesting to note the value of ϵ just determined. The Berman value of $\epsilon = 0.30$ is about the average of most values determined by other investigators (Ref. 3). Although the value $\epsilon = 0.77$ is somewhat higher, it should be noted that:

(1) ϵ tends to take on higher values when evaluated over regions close to the Sun ($\sim 10 R_0$)

(2) Spectral broadening is probably proportional to more than just the signal path electron content.

Saito, for instance, obtains a value of $\epsilon = 0.5$ for the region 1 to 5 solar radii (Ref. 4).

In his paper, Woo states that the bandwidth is proportional to the solar wind flux. Assuming a spherically symmetric corona would imply that the bandwidth falls off as an inverse-square law. For completeness, a simple inverse-square function was fit in the least-squares sense to the bandwidth data. This fit yielded the statistics

$$\sigma(\text{dB}) = 2.0024$$

A plot of this function is seen in Fig. 4.

V. Conclusion

In its final form, the empirical model for the spectral bandwidth during superior conjunction is given by

$$BW(\alpha) = K(\text{ISED})$$

$$= (1.95 \times 10^{-1}) \left[\frac{\pi - \alpha}{(\sin\alpha)^{1.77}}\right] F(\alpha)$$

$$+ (4.2 \times 10^{-9}) \left[\frac{1}{(\sin\alpha)^5}\right], \text{ Hz}$$

with

$$F(\alpha) = 1.0 - 0.13 \left[\frac{(\pi/2)^3 - (\alpha - \pi/2)^3}{\pi - \alpha}\right]$$

$$+ 0.002 \left[\frac{(\pi/2^5) - (\alpha - \pi/2)^5}{\pi - \alpha}\right]$$

$$\alpha = \text{SEP angle, rad}$$

This expression is valid for the region $1R_0$ - $20R_0$

This bandwidth can be expressed in different units using the following conversion factors

$$5600 \ \frac{\text{meters}}{\text{Hz}}$$

$$1.95 \times 10^{20} \ \text{el/m}^2/\text{Hz}$$

The model presented gives the user a best-fit expression for the spectral bandwidth as a function of heliocentric distance during superior conjunction. The ISED function appears to reflect the basic signature of spectral broadening effects in the region $1R_0$ - $20R_0$.

Although spectral broadening is undoubtedly a result of more complicated processes than just the presence of electrons in the signal path (or even the fluctuations in the density of these electrons), ISED, with ϵ = 0.77, describes the radial behavior of this effect to a good first order. Perhaps this model will be a good starting place for a more detailed description of solar corona spectral broadening effects.

References

1. Woo, R., Yang, F., and Ishimaru, A., "Structure of Density Fluctuations Near The Sun Deduced From Pioneer-6 Spectral Broadening Measurements," in *The Astrophysical Journal*, Vol. 210, No. 2, Part 1, December 1, 1976.

2. Berman, A. L., and Wackley, J. A., "Doppler Noise Considered as a Function of the Signal Path Integration of Electron Density," in *The Deep Space Network Progress Report 42-30*, Jet Propulsion Laboratory, Pasadena, California, June 15, 1976.

3. Berman, A. L., Wackley, J. A., Rockwell, S. T., and Kwan, M., "Viking Doppler Noise Used to Determine the Radial Dependence of Electron Density in the Extended Corona," in *The Deep Space Network Progress Report 42-38*, Jet Propulsion Laboratory, Pasadena, California, January 1977.

4. Saito, K., "A Non-Spherical Axisymmetric Model of the Solar K Corona of the Minimum Type," *Annals of the Tokyo Astronomical Observatory*, University of Tokyo, Second Series, Vol. XII, No. 2, Mitaka, Tokyo 1970.

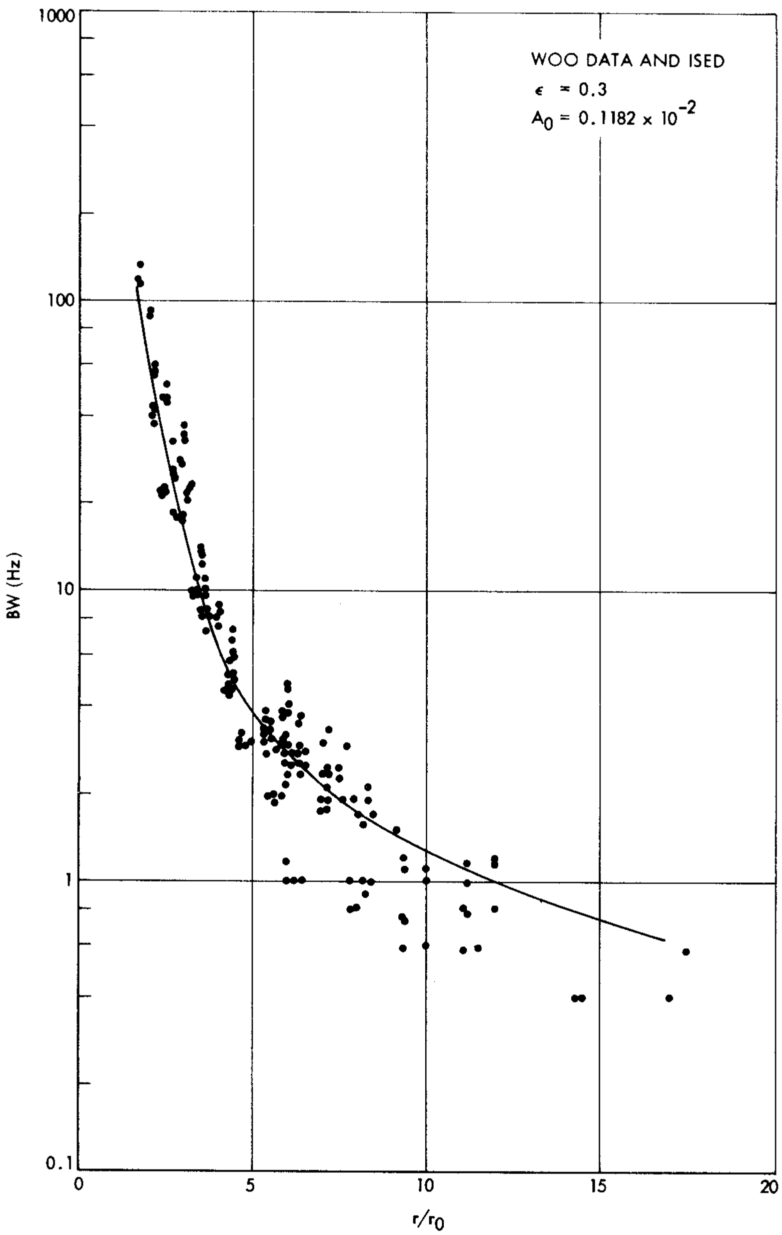

Fig. 1. Woo data and ISED, $\epsilon = 0.3$ and $A_0 = 0.1182 \times 10^{-2}$

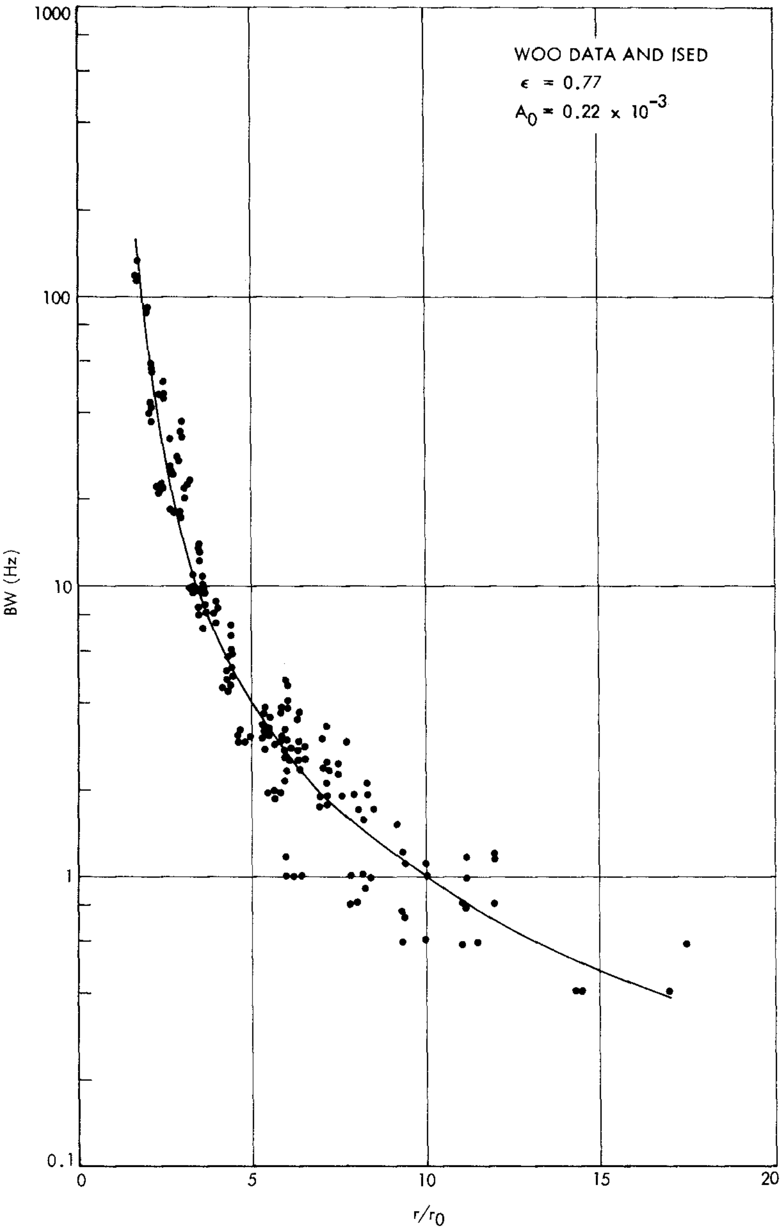

Fig. 2. Woo data and ISED, $\epsilon = 0.77$ and $A_0 = 0.22 \times 10^{-3}$

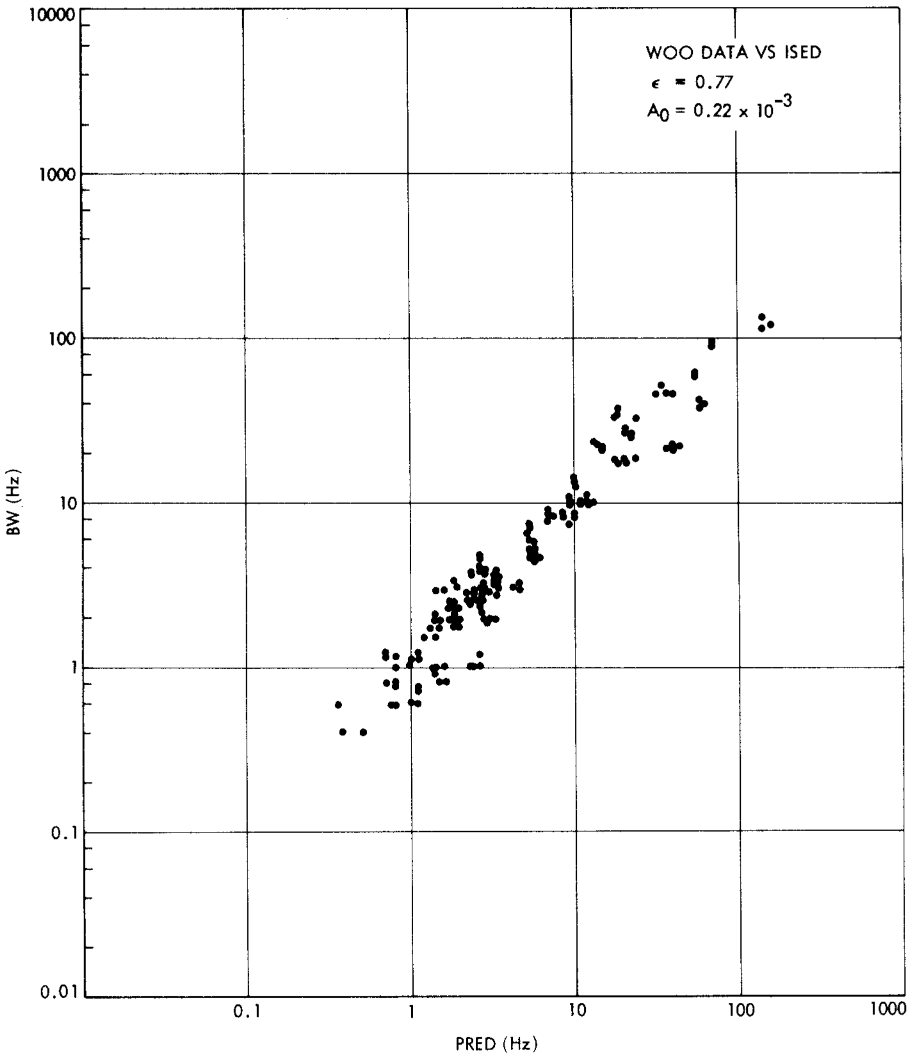

Fig. 3. Woo data vs ISED, ϵ = 0.77 and A_0 = 0.22 × 10^{-3}

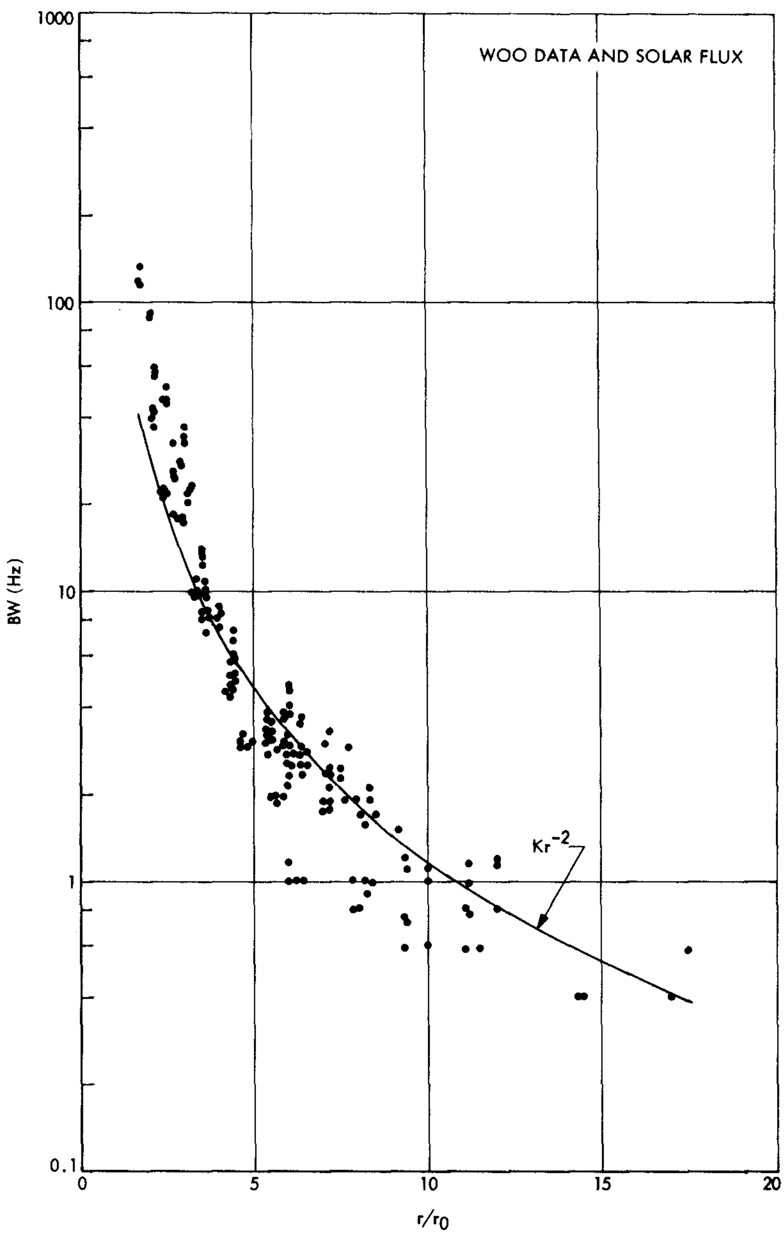

Fig. 4. Woo data and solar flux